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ABSTRACT

This paper provides for the elementary school teacher an introduction to the problem of mathematics readiness. The first part outlines the four developmental stages as seen by Piaget, and the manner in which schemata develop through the process of assimilation and accommodation. The second part consists of seven Piagetian tasks which a teacher may use to assess a child's stage of development. (MM)

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MATH READINESS
and
RELATED CONSIDERATIONS

1972

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and
RELATED CONSIDERATIONS

by
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ERDC Research Assistant

March, 1972

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LETTER OF TRANSMITTAL

Dear Council Member:

Educators have long recognized that success in learning is closely related to the degree of readiness possessed by the learner to deal with the task at hand. This publication has been developed at the request of our membership to provide needed insights into the concept of readiness as it applies to mathematics.

Special appreciation is extended to Michael A. Appleman, ERDC Research Assistant and writer of the project.

Sincerely yours,

Thomas F. Stark

Thomas F. Stark
Executive Secretary

TFS/rrb

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MATH READINESS

INTRODUCTION

Educational historians will look back upon the twentieth century and will no doubt characterize the sixties as the decade for curriculum reform.[1] During this time, many changes reshaped, refocused, and radically altered every subject in the elementary curriculum. Mathematics was the first area to undergo major revision. Reacting to real or imagined challenges from the Russian Sputnik in 1957, many Americans criticized our apparant scientific progress and demanded more rigorous school programs.[2]

Responding to public indignation, Congress, in 1958, passed the National Defense Educational Act (NDEA). It also increased appropriations to the National Mathematics Foundation to support new curriculum projects and to update the mathematics background of teachers.[3]

These efforts to change mathematics education were so drastic, so vivid, and so well publicized that they are often referred to as the "mathematics revolution." Especially for the layman, the new vernacular and program components were so different from his own schooling that it was indeed an overthrow of the old system, "a revolution." To educators, however, it was more accurately an "evolution." The foundation and basic premises had evolved from previous curriculum movements.[4]

One indication of the rapidity of change in the mathematics curriculum is reflected in the concept of "math readiness." When is a child ready to learn mathematics? A review of the literature indicates that teachers are attempting to teach certain mathematical concepts to elementary youngsters before the child's level of development allows desired learning to take place. In short, it is argued that learning about numbers,

including the basic addition facts and the concept of place value should be deferred until the child indicates math readiness on the basis of recognized diagnostic tasks.

STATEMENT OF THE PROBLEM

This paper provides an introduction to the problem of math readiness. Much research has shown that readiness for mathematics is not just a function of age and intelligence. It is also a function of the child's "developmental stage." The first purpose of this paper is to provide an understanding of Piaget's theory of developmental stages. The second purpose is to provide a Piagetian task administration instrument which will aid the mathematics teacher in determining a child's stage in development. Thus, the instrument provides the teacher with valuable insights into a child's readiness for math.

PIAGET'S THEORY OF DEVELOPMENTAL STAGES RELATED TO MATH READINESS

Jean Piaget, the Swiss psychologist, has contributed a wealth of material to psychology and learning theory. Twenty years ago he received not a single mention in the yearbook on mathematics education.[5] Today it is nearly impossible to discuss the psychology of instruction in mathematics without placing Piaget's contribution at center stage. His influence is not limited to the psychology of instruction. "Many psychologists are seriously suggesting that his stature will eventually equal that of Freud as a pioneering giant in the behavioral sciences."[6]

The classroom teacher, however, has little opportunity to examine Piaget's theory of developmental stages and then implement his ideas into the curriculum. At the same time most teachers agree that they could benefit

from a deeper understanding of a child's level of development as it applies to teaching. Thus, this paper is designed to put theory into practice.

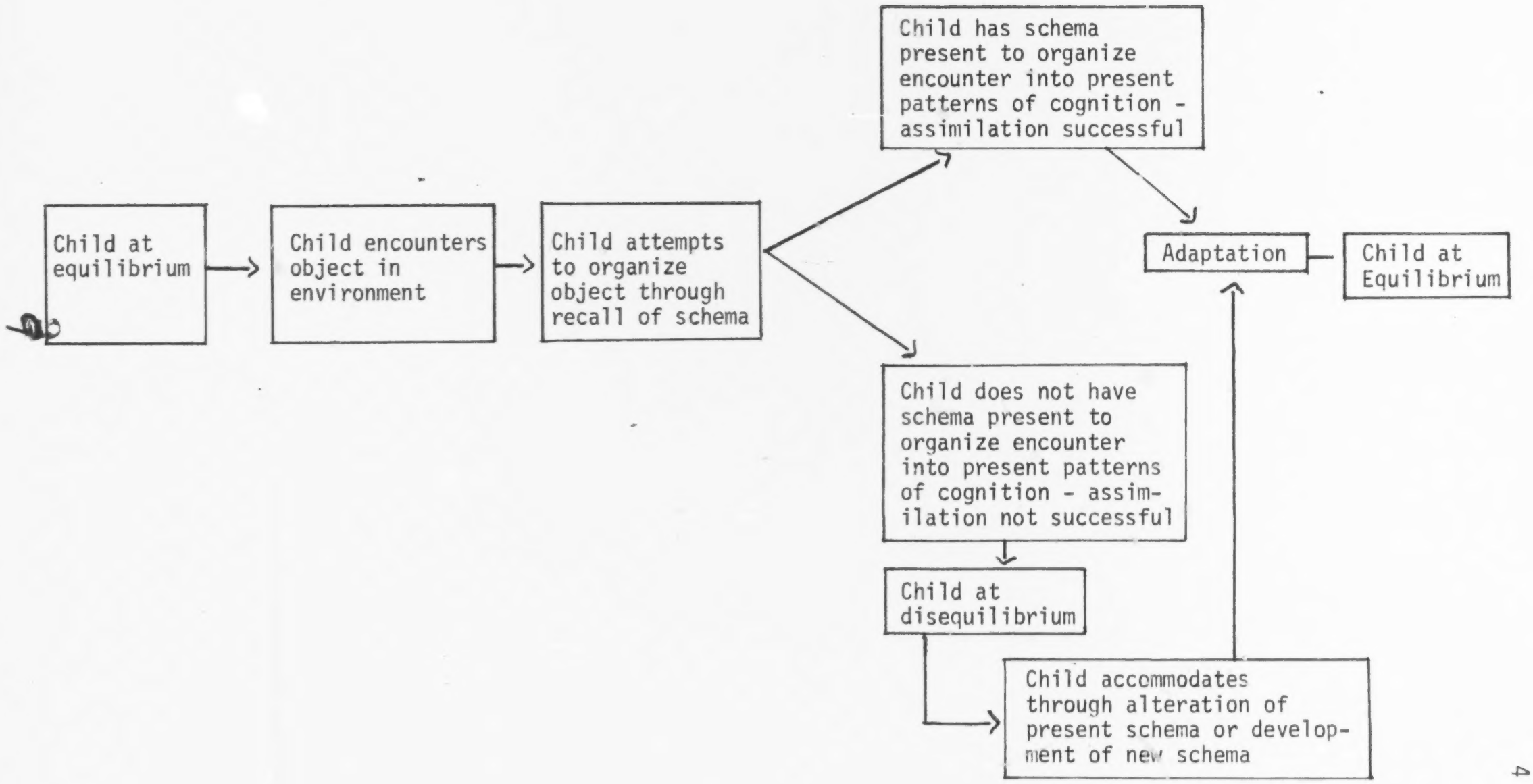
Piaget has designed and administered tasks probing the developmental stages of thousands of children. Based on the accumulated evidence, he has theorized a heirachy of stages in the growth of the cognitive ability in children.

Intelligence, its acquisition and utilization, is basic to the model of development by Piaget. He states, "intelligence is a continuing process of adaptation and organization as the child interacts with the physical and social environment."[7] (See Figure 1, page 4) The organization of a child's intelligence is based on schemas. These are the intellectual "structure," "units," or "programs," that can be recalled by the child to understand objects with his environment. The process of adaptation is divided into assimilation and accommodation. Assume the child is at equilibrium with his environment. Then, he encounters an unfamilar object. If he is able to remember schemas and can organize this object into present patterns of behavior, assimilation is successful; the child has adapted and remains at equilibrium. If the object cannot be assimilated, however, a disequilibrium occurs and either old schemas are modified or new schemas are developed. Accomodation has occurred in the child's cognitive development. In the Piagetian model, intelligence is an evolutionary process. Thus, evolution moves toward an equilibrium between assimilation and accomodation.[8]

In view of this description of intelligence, Piaget has formulated a series of developmental stages. The stages of development in children are "sensory-motor," "preoperational," "concrete operations," and "formal operations."[9] As outlined above, in respect to intelligence in Piaget's model is a set or composite of schemas that, at a given time, is at relative

COGNITIVE DEVELOPMENT
AS VIEWED BY JEAN PIAGET

Figure 1


Child at equilibrium
Child encounters object in environment
Child attempts to organize object through recall of schema
Child has schema present to organize encounter into present patterns of cognition - assimilation successful
Adaptation
Child at Equilibrium
Child does not have schema present to organize encounter into present patterns of cognition - assimilation not successful
Child at disequilibrium
Child accommodates through alteration of present schema or development of new schema

equilibrium in the child's development. As operation has been defined by Piaget as "an action that can be returned to its starting point, and that can be integrated with other actions possessing this feature of reversibility."[10] John Flavell has also defined operation as "any representational act that is an integral part of an organized network of related acts. . ."[11] In short, if a child can make mental transformation on objects, he can also reverse the transformation and conserve the total process.

Sensory-Motor Stage

The sensory-motor stage begins at birth and lasts approximately eighteen to twenty-four months. This is a pre-verbal stage; the child increases in ability to organize or coordinate his physical actions. During this stage the child deals only with real objects; he is unable to produce mental operations without physically acting upon the object being thought about. Likewise, there is no conception of object permanence. If the object is removed or hidden, as far as the child is concerned, the object does not exist. He has no permanent record of the object. The basic principles of practical knowledge are being acquired. Primitive concepts about space, time, casuality and intentionality are within the cognitive structure of the child by the end of the sensory-motor period.[12]

Preoperational Stage

As this stage implies, the child cannot complete operations as defined by Piaget. The child is unable to conserve, which is a manifestation of the ability to reverse thought processes or operations. During the early part of the preoperational stage the child's logical reasoning is often transductive (particular to particular) as opposed to inductive (particular to general) or deductive (general to particular). There is a gradual evolution of

centrism, i.e., the ability to focus on a problem or situation in the pre-operational child. Early in the period the child will center on only one dimension of a problem; for example, either length or width, but not both. In the latter part of the stage the single centering has evolved to two successive centerings; for example, first length then width, but the child cannot center on both dimensions concurrently. Memory is developing. The child has the ability to think of the past and the future; however, he cannot think very far in either direction. As the child investigates his world, counter intuitive events force accommodation while repetition of activity through involvement results in assimilation.[13]

Concrete Operations

The child can perform mental operations; however, they must be performed on physical objects. Logic is rational, as perceived by adults. Conservation of quantity, weight, and volume are possible. Also, they generally emerge in the child's logical thought processes in the order outlined above. Verbal justification is possible for the conservation problems; the child will justify the conservation through reversibility. The ability to hold two centerings simultaneously has developed. Through a recall of schemas or organized ideas the child can organize his environment. He can arrange events in historical order. Mental transformations between states are possible. Thinking is more objective. During the preoperational stage the child's thinking is very egocentric. Now he is able to observe, judge, and evaluate in less egocentric terms. The child can think of properties as a ground, establish action-reaction relationships and mentally represent several related actions simultaneously. Still, however, the child cannot verbally express an hypothesis following a long series of related ideas. If given different objects or pictures, he can form simple classes; however, he

cannot group classes into more comprehensive groups.[14]

Formal Operations

Mental operations are possible. No longer is there a need for objects before operations can be completed. Thoughts are stimulus free. In the prior stage thoughts were related to a specific stimulus usually from the perceived environment. The child's thoughts and ideas can originate from within. Ideas can be generated relative to other ideas. The individual can infer and form theories about the future based on experiences of the past. He can then consider alternatives to those theories. In short, he has developed the cognitive ability to intellectually manipulate hypothetical situations and then systematically evaluate many alternatives relative to the hypothetical situation. At this stage probability is also understandable.[15]

IMPLICATION OF PIAGET'S THEORY OF DEVELOPMENTAL STAGES

The preceding discussion of Piaget's theory of developmental stages indicates that the child's view of space, volume, and matter is not an adult's view. The adult mind has the benefit of a variety of experiences and has developed a consistent mental framework through which it processes encounters with the environment. "The child, when confronted with a new problem, will often reason inconsistently, even logically."[16] Elementary mathematics teachers should realize this. Due to a lack of understanding of Piaget's theory of developmental stages, however, math teachers all too often have tried to teach concepts requiring mental processes exceeding the pupil's level of cognitive development. In short, it is likely that many teachers are attempting to teach mathematical concepts before a child can understand them. Attempting to learn certain mathematical concepts, therefore, should be deferred until children can demonstrate readiness by

successful mastering of diagnostic tasks based on Piaget's theory of developmental stages.

DIAGNOSTIC TASKS TO ASSIST IN DETERMINING MATH READINESS

The following Piagetian tasks were developed at Colorado State College, Greeley, Colorado. Many modifications of these are possible and applicable to the in-depth probing of a child's understanding of a specific mathematical concept. A very important aspect of the interview used in administering these tasks is the child's justification of his response.[17] This will give insight into the ability of the child to reverse his thinking and thus complete the operation. It is also important that the child realize that he is not being evaluated in a pass-fail test. The child's response is always correct. Whether it is logical to an adult thinking, simply indicates the level of development.

Piagetian Task Administration Instrument

Interviewer _____ Child _____
Sex (M or F)

Date _____ Age _____

School _____ Grade _____

TO THE INTERVIEWER: Following each task there is a box to be used for evaluating the task. If the child has performed the task in a manner that would be correct to an adult's mind, i.e., the child was able to conserve mass or a liquid, this is indicated by placing an + (plus) in the box. If the child indicates an incorrect response, place a 0 (zero) in the box. These will be tallied at the end of this interview sheet.

Experiment 1 - Conservation of Quantity

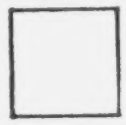
Present the child with a ball of clay. Ask the child to observe it. Then roll the clay into a long cylinder. Ask - "Does the snake have less, more, or the same amount of clay as the ball?" (If the child is confused say, "Was the ball bigger, smaller, or the same size as the snake?"

RESPONSE

LESS MORE SAME

JUSTIFICATION - Ask the child, "why do you think the snake was (bigger, smaller, or the same)?"

TASK 1



Experiment 2 - Conservation of Liquid Volume

Use two jars (a baby food jar and a tall cylinder) and enough colored water to fill the tall jar. Present the jars with the water in the short jar. Ask - "What will happen if I pour the water into the tall jar? Will I have more, less, or the same amount of water?"

RESPONSE

LESS MORE SAME

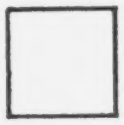
Pour the water into the jar. Ask - "What happened to the amount of water? Is there less, more, or the same amount?"

RESPONSE

LESS MORE SAME

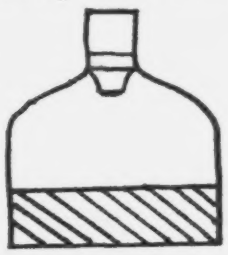
JUSTIFICATION - Ask the child, "Why do you think the amount of water was (more, less, the same)?"

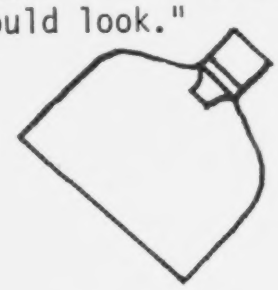
TASK 2



Experiment 3 - Conceptualization of Water Level

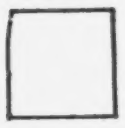
Ask the child to look at the drawing of the jar. Tell the child the jar has water in it. (Point to water) Also tell him the jar is plugged. (Point to plug) Ask "if the jar were tipped as you see in this picture, how would the water look? Make a line with your pencil showing how the water would look."





JUSTIFICATION - Ask the child, "Why do you think the water will look like that?"

TASK 3

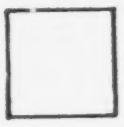


Experiment 4 - Ordering Events

Say - "For this problem, you will think about how a pencil falls. This is what I mean: (place a pencil in a vertical position and allow it to fall to a horizontal position on the desk). Here are some drawings of the pencil falling. Place them in order showing how the pencil would look as it falls." (See Figure 2, page 14)

JUSTIFICATION - Ask the child, "Why did you place the pictures in the order you did?"

TASK 4



Experiment 5 - Displacement of Volume

Use a tall cylinder three quarters (3/4) full of colored water and two metal blocks of the same size (volume) but different weights.

Tell the child to compare the weights of the two blocks. Hand him the blocks and ask, "Which is heavier?"

Say, "If I take the light weight block and lower it into the water, what will happen to the level of the water?"

RESPONSE

HIGHER LOWER SAME

Ask the child, "Place the rubber band around the cylinder at the level you think the water will move to." You may wish to aid the child by holding the cylinder or helping with the rubber band.

Gently lower the block into the water and observe. If necessary, move the rubber band to the level of the water at this point. Then remove the block.

Ask the child, "Where do you think the level of the water will be when the heavier block is lowered into the cylinder? Will the water level for the heavier block be lower, higher, or the same as the water level for the lighter block?"

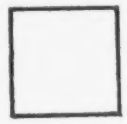
RESPONSE

HIGHER LOWER SAME

JUSTIFICATION - Ask, "Why do you think this will happen?"

Lower the heavier block into the cylinder and observe. List the child's comments about result.

TASK 5



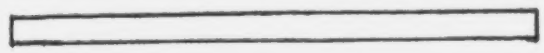
Experiment 6 - Conservation of Length

Use a complete and a sectioned straw. Start both straws lined up parallel. Note with the child that both straws are the same length. Ask, "Would two ants starting a hike at this end of the straws (point to one end) and walking at the same speed both finish the hike at this point (point to other end of straws) at the same time?" (If the child is confused, ask, "Would they both travel the same distance?"

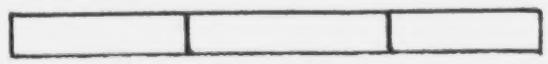
RESPONSE

YES NO

Now move the whole straw into this position



Ask the same question.

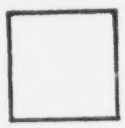


RESPONSE

YES NO

JUSTIFICATION - "Why do you think so?"

TASK 6a



Now move the straws into this position.

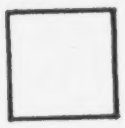
Ask the same question.

RESPONSE

YES NO

JUSTIFICATION - "Why do you think so?"

TASK 6b

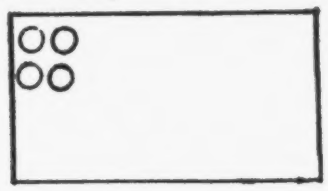


Experiment 7 - Conservation of Area

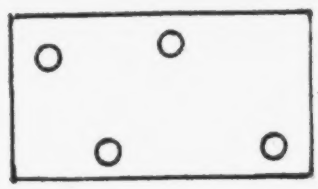
Present the child with two identical pieces of green construction paper. Tell him these represent fields or pastures. Place one animal on each piece of paper.

Ask the child to compare the fields noting they are the same size. Comment that since the fields are the same size each animal will have the same amount of grass to eat. Tell the child you are going to use stoppers to represent barns.

Place four barns on each field as shown. (Leave the animals on the field.)



A - Clustered Barns



B - Barns Spread Out

Ask, "Now which animal will have the most grass to eat or will the amount of grass be the same?"

RESPONSE

Field A Field B Same

JUSTIFICATION - "Why do you think this is true?"

Continue adding equal numbers of barns to each field. Each time repeat the question, "Which animal will have the most grass to eat?"

COMMENTS:

TASK 7

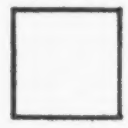
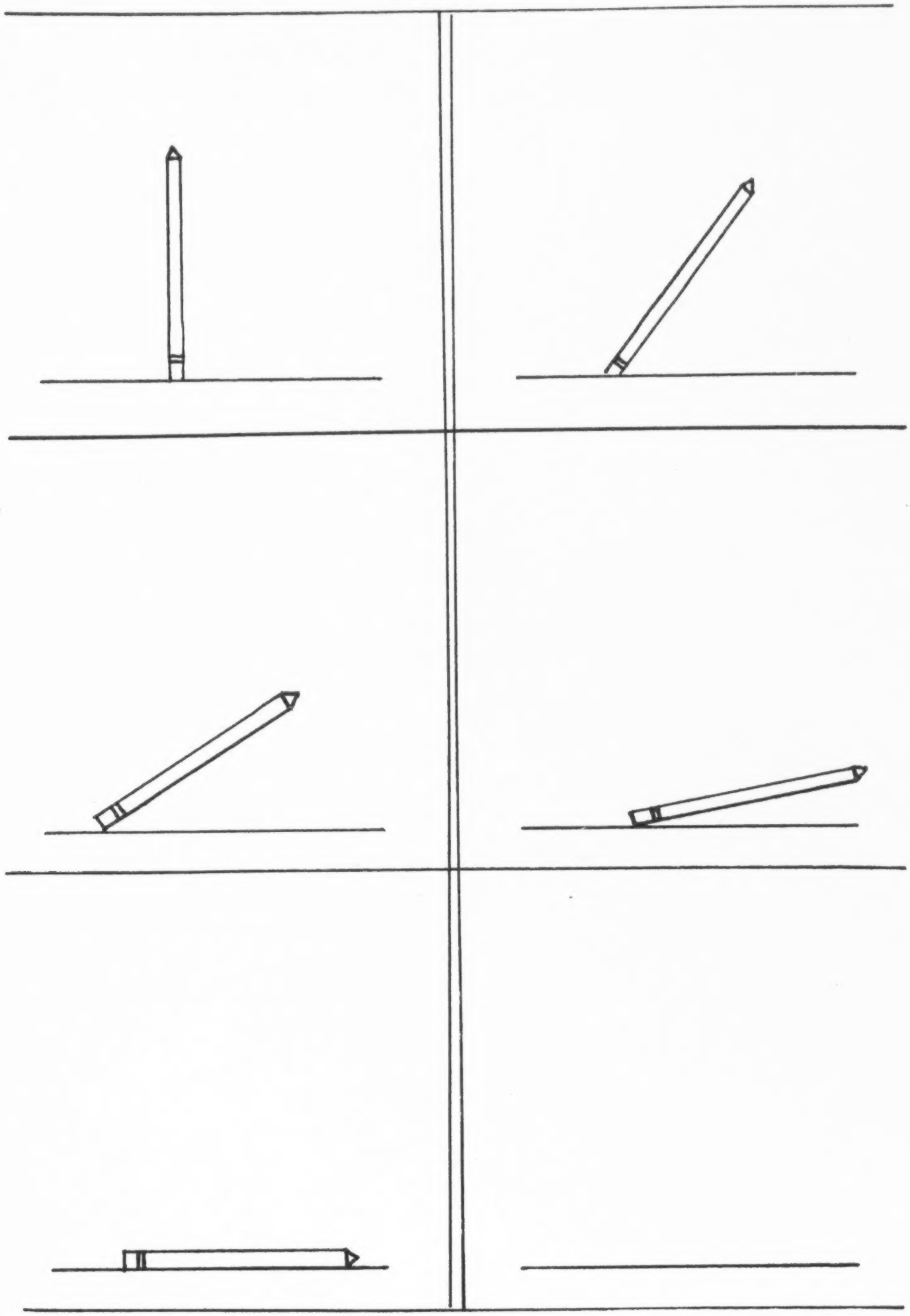


FIGURE 2



SUMMARY AND ANALYSIS OF TASKS

	Evaluation of Task	Cognitive Level
TASK 1		
TASK 2		
TASK 3		
TASK 4		
TASK 5		
TASK 6a		
TASK 6b		
TASK 7		

At which level of cognitive development would you place this child? (0-2 tasks performed - pre-operational; 3-6 - transitional; 7-8 - concrete operational)

☐ Pre-operational

☐ Transitional

☐ Concrete operational

Additional comments or observations:

SUMMARY

Through greater understanding of students' developmental stages, the mathematics teacher is obviously in a better position to determine whether a child is ready for certain mathematical concepts. In addition, grouping for activities could center around the developmental stage of the child. These tasks also can identify some problems and needs of the "slow learner."

This paper provides an introduction to the theory and work of Piaget. More vital, however, it can be noted that his theory is applicable to the elementary mathematics teacher in his everyday work. It seems, however, that the greatest difficulty is bridging the gap between theory and practice. It is hoped that this paper has provided some direction and ideas for joining the two.

Teachers should always attempt to understand their students. Administering the Piagetian task instrument is one method of achieving that goal. Through better understanding of the individual, there is an approach to the essence of individualized instruction.

FOOTNOTES

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[2]Ibid., p. 1.

[3]Ibid., p. 2.

[4]Ibid., p. 3.

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[8]Ibid., p. 1.

[9]Copeland, Richard W. How Children Learn Mathematics. New York: The Macmillan Company, 1970, p. 14.

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[13]Ibid., p. 2.

[14]Ibid., p. 3.

[15]Ibid., p. 4.

[16]Ibid., p. 5.

[17]Ibid., p. 5

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